Possible Solutions to the 2013 AP Statistics Free Response questions.

Some notes:

- Please make critiques, suggest improvements, ask questions. This is just one AP stats teacher's initial attempts at solving these. I, as you, want to learn from this process.
- I simply construct these as a service for both students and teachers to start discussions. There is nothing "official" about these solutions. I can't even guarantee that they are correct. They are simply one statistics teacher's attempt at the problems. I do this as a way to invite dialogue about the questions.
- Go to the following public site to access the 2013 problems. Questions and scoring rubrics for previous tests are also at this link. http://apcentral.collegeboard.com/apc/members/exam/exam_information/8357.html

1.

a) "Unhealthy" means that the lead level is greater than 6.0 ppm. In the stem plot, four of these had levels above 6.0 ppm (6.3, 6.4, 6.6, and 6.8 ppm). Therefore the proportion of the sample of crows classified as unhealthy by the biologist is $\frac{4}{23} \approx .1739$.

b) Setup: I will construct a 95% confidence interval to estimate μ_X = mean lead level of all crows in the region. Procedure: 1 sample *t* interval for μ_X, df = 22.

Conditions: The problem states that a random sample of crows was selected (Randomness condition met). The lead levels appear unimodal and roughly symmetric, so we feel confident that the underlying sampling distribution of $\overline{X}$ is approximately normal. We're sampling crows from the population without replacement. As long as the population of crows is >230, then we have 23 < (0.1)(230), and outcomes from crow to crow are roughly independent.

Mechanics: $\overline{x} \pm (t^{*}_{22})(\frac{s_x}{\sqrt{n}}) = (4.90) \pm (2.074)(\frac{1.12}{\sqrt{23}})$ My interval is (4.4157, 5.3843).

Conclusion: With 95% confidence, I estimate that the mean lead level for all crows in the region is somewhere between 4.416 ppm and 5.3843 ppm.

2.

a) If the first 500 students who enter the stadium are different than the population of all students with their satisfaction levels, then our estimate may be biased. For example, perhaps the first 500 students who enter the stadium tend to live in "below average" dorms and head to the game early to get out of their shoddy living conditions. This group would probably be less satisfied than a more representative sample. A satisfaction rate computed from such a sample is likely to produce an estimate of the satisfaction rate that is probably too low.

b) Alphabetize the list. Then number each student from 1 – 70,000 (First student alphabetically = 1, second alphabetically = 2, … last alphabetically = 70,000). Use a random number generator to select 500 different integers from 1-70,000 (no repeats). The students from our alphabetized list with those numbers are in our sample.

c) Stratification by campus would be better than stratification by gender ***if the association between campus and satisfaction is stronger than the association between gender and satisfaction.*** Suppose, for example, that satisfaction rates between males and females are identical (i.e.: that gender and satisfaction are independent). Then stratifying by gender won't increase the precision of our estimate over an SRS much at all. But if people on the first campus love the grounds and those on the second campus don't, then satisfaction rates within these two groups will be different. We can capitalize on this difference in satisfaction rates by getting a sufficient number of students from each campus, calculating a satisfaction rate for each stratum, and then using those satisfaction rates to compute an appropriate estimate of the proportion of all students who are satisfied (use an appropriately weighted average of the two satisfaction rates). This estimate will vary less than the ones from sample stratified by gender.

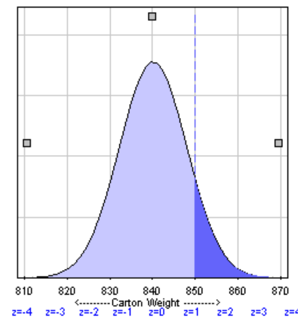
3.

Let’s organize the information in this problem:

X = weight of a randomly selected egg.

C = weight of a randomly selected empty cardboard container

T = weight of a full carton. Notice that $T = X_1 + X_2 + \ldots + X_{12} + C$.



a) $P(T > 850) = P(Z > \frac{850-840}{7.9}) \approx P(Z > 1.27) \approx .1028$. See the picture.

b)

i.- Because $T = X_1 + X_2 + \ldots + X_{12} + C$, we can use this fact to compute the following: $E(X_1 + X_2 + \ldots + X_{12}) = E(T - C)$

Note, therefore that $E(X) = \frac{1}{12} E(X_1 + X_2 + \ldots + X_{12}) = \frac{1}{12}(E(T) - E(C))$

This gives us $E(X) = \frac{1}{12}(840 - 20) = 68.333\ldots$ grams.

ii. $X_1 + X_2 + \ldots + X_{12} = T - C$. Since we can assume independence between weights of eggs and the weights of the containers, we can compute the following:

$$\sigma_{X_1+X_2+\ldots+X_{12}} = \sigma_{T-C}$$

$$\sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2 + \ldots + \sigma_{X_{12}}^2} = \sqrt{\sigma_T^2 + \sigma_C^2}$$

$$\sqrt{12}\sigma_X = \sqrt{\sigma_T^2 + \sigma_C^2}$$

$$\sigma_X = \frac{\sqrt{\sigma_T^2 + \sigma_C^2}}{\sqrt{12}} = \frac{\sqrt{(7.9)^2 + (1.7)^2}}{\sqrt{12}} \approx 2.3327 \text{ grams.}$$

4.

Setup: We will perform a χ^2 test of association. The hypotheses are:

H_0 : There is no association between age group and whether or not one consumes five or more servings of fruits and vegetables per day for all adults in the US.

H_A : There is an association between age group and whether or not one consumes five or more servings of fruits and vegetables per day for all adults in the US.

Let's use $\alpha = 0.05$.

Conditions:

Large sample size: By checking the expected counts, we see that they are all >5.

Independence: We have a random sample of all adults in the US. Furthermore 8866< 10% all adults in the US (there are millions)

Randomness: A random sample of adults in the US were selected.

Mechanics:

Observed (expected)	Yes	No	Total
Ages 18-34	231(240.204)	741 (731.795)	972
Ages 35-54	669(719.377)	2242(2191.622)	2911
Ages 55 or older	1291(1231.418)	3692(3751.58)	4983
Total	2191	6675	8866

$\chi^2 = \sum \frac{(obs-\exp)^2}{\exp} \approx 8.983$. *Df* = 2. P-value = 0.0112

Conclusion: Because our p-value < 0.05, we will reject the null hypothesis at the 5% level. We have evidence that there is an association between age group and whether they eat more than 5 servings of fruits/vegetables a day. In particular, Those in the 55+ age group are more likely to report "yes" to eating fruits/vegetables than those in the younger age groups.

5.

a) No. Because the daily meditation was not imposed randomly on each subject, we cannot claim that the meditation is responsible for the lower rates of high-blood pressure in that group. It may be that those who meditated were also consuming less sodium than the non-meditators. The difference in sodium levels and not the meditation may be the reason for the lower rates of high blood pressure.

b) Because the number of successes (has high blood pressure) and failures (does not have high blood pressure) within each treatment group were not both greater than 10:

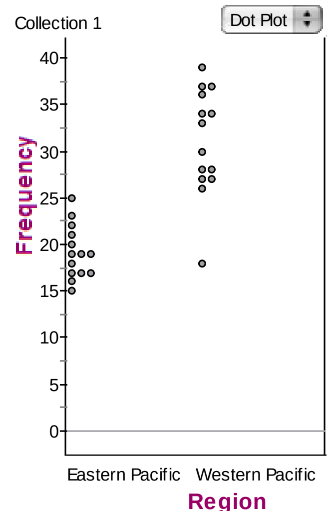
Group	# successes (with high BP)	# failures no high (BP)
Meditation group	0	11
Control Group	8	9

Three of these four values are less than 10. So the sampling distribution of $\hat{p}_m - \hat{p}_c$ cannot be assumed to be normal.

c) In our samples, $\hat{p}_m - \hat{p}_c = \frac{0}{11} - \frac{8}{17} \approx -0.47$. To make a conclusion based on this evidence, we need to see how likely it is to get a value for $\hat{p}_m - \hat{p}_c$ as low/ lower than -0.47 by chance if we run a simulation where we assume that null hypothesis is true $p_m - p_c$. A sample result this low/lower happened in 76/10,000 samples: A simulated *p*-value would then be about .0076 this is lower than .01, So we have evidence at the 1% level that men who meditate in this retirement community have lower rates of high BP than those who do not meditate. However, we cannot claim that the meditation is the cause of the lower rates of high BP.

6.

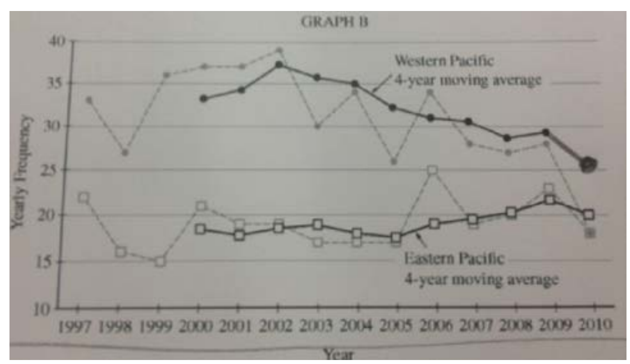
a) The dot plot to the right helps justify my comparison: More typhoons occur in the Western Pacific each year than in the Eastern Pacific, on average. The variability of "# typhoons" from year to year is higher in the Western Pacific than in the Eastern Pacific. Finally, the typhoon frequencies have different shapes in the two regions: In the Eastern Pacific, the distribution of typhoon frequencies appears unimodal and roughly symmetric. In the eastern pacific, frequencies appear a bit skewed left: The year with the lowest # typhoons in the Western Pacific may be an outlier, however.



b) In the Western Pacific, see an overall decrease in the number of typhoons per year from 1997-2010. However, between 1997-2002, the number of typhoons increased. From 2003- 2010 it seems to have decreased. But in the Eastern Pacific, the number of typhoons between 1997-2010 seemed a be a bit more steady – perhaps a slight increase over this time period.

c) $\frac{28+27+28+18}{4} = 25.25$ typhoons/year over the years 2007-2010.

d) See the picture: The chart is completed.



e-i) The plot with the 4-year moving makes the long-term trends of how the number of typhoons has changed more apparent. It's now easier to see that the # typhoons has decreased from 2000-2010 in the Western Pacific, but very slightly increased in the Eastern Pacific. This is due to the fact that averages tend to be less volatile (less noisy) than individual measurements. The year-to-year variation in the previous plot obscured this trend.

e-ii) The plot of 4-year moving averages make it harder to observe how unpredictable the number of typhoons is from year-to-year. The plot of the *average* # typhoons in the past 4 years show much less volatility than the plot showing the *individual* number of typhoons from year to year. The moving averages may make the degree of unpredictability from year to year less apparent.